Progress in Astronautics and Aeronautics: Celestial Mechanics and Astrodynamics, Vol. 14, edited by V. G. Szebehely, Academic Press, New York, 1964, pp. 35–51.

[2] Stiefel, E., "Report to NASA on Methods of Regularization for Computing Orbits in Celestial Mechanics," NGR52-056-001, 1966, Institute for Applied Mathematics of the Swiss Federal Institute of Technology, Zurich, Switzerland.

[3] Tapley, B. D., Szebehely, V., and Lewallen, J. M., "Trajectory Optimization Using Regularized Variables," *AIAA Journal*, Vol. 7, June 1969, pp. 1010–1016.

[4] Lewallen, J. M., Schwausch, O. A., and Tapley, B. D., "Coordinate System Influence on the Regularized Trajectory Optimization Problem," AIAA Paper 69-903, Princeton, N.J., 1969.

[5] Czuchry, A. J. and Pitkin, E. T., "Regularization in Optimal Trajectory Problems," *AIAA Journal*, Vol. 6, No. 6, June 1968, pp. 1209–1210.

[6] Sperling, H. J., "The Collision Singularity in a Perturbed Two-Body Problem," *Celestial Mechanics*, Vol. 1, Oct. 1969, pp. 213–221.

[7] Pitkin, E. T. and Carpenter, G. C., "Modified Newton-Raphson Methods for Preliminary Orbit Determination," *Celestial Mechanics*, Vol. 1, June 1969, pp. 72–90.

Some Simple Scaling Relations for Heating of Ballistic Entry Bodies

MICHAEL E. TAUBER*
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

A = base area of entry body
B = ballistic parameter, see Eq. (3)
C = constant, see Eq. (4)
C_D = drag coefficient
k = constant equal to $B/2(M - 1)$
m = body mass
M = constant, see Eq. (4)
N = constant, see Eq. (4)
q = heat transfer into the body per unit area
Re = Reynolds number
t = time
V = flight velocity
y = flight altitude
β = inverse scale height of atmosphere
γ = flight-path angle below horizontal
ρ = freestream density

Subscripts

E = entry
F = final
0 = reference value (for Earth — sea level)

BALLISTIC atmospheric entries can be characterized by three parameters. These describe the body, the flight path, and the atmosphere; they are the ballistic coefficient m/C_DA, the flight-path angle γ, and the inverse atmospheric scale height β, respectively. In this Note some scaling relations are derived to show the influence of the aforementioned three parameters on the total heating, per unit area, of a body making a ballistic entry.

During that part of the entry where aerodynamic heating is important, the drag is generally much greater than the weight component along the flight path and the equation of motion can be written

$$\tfrac{1}{2}\rho V^2 C_D A \doteq -m(dV/dt) \tag{1}$$

If the flight-path angle is not too shallow ($\gamma_E \geq 8°$, approximately, at the beginning of the sensible atmosphere), it can be assumed that

$$\sin\gamma \doteq \text{const} \doteq \sin\gamma_E \tag{2}$$

Using the assumption of an exponential atmospheric density variation with altitude, Allen and Eggers[1] integrated the equation of motion and showed that

$$V/V_E = e^{-(B/2)\bar{\rho}} \tag{3}$$

where

$$B = (C_D A/m)(\rho_0/\beta \sin\gamma_E) = \text{const}$$

and

$$\bar{\rho} = \rho/\rho_0$$

Now, it is assumed that the heating rate per unit area can be written in the form

$$(dq/dt) = C(\bar{\rho})^N V^M \tag{4}$$

where N, M, and C are constants. Equation (4) is a reasonable approximation for both laminar[2] ($N = 0.5$, $M \doteq 3.1$) and turbulent[3] ($N \doteq 0.8$, $M \doteq 3.7$) convection if ablation product blockage is negligible. For radiation, Eq. (4) is only a crude approximation; for continuum air radiation $N = 1.8$ and $M = 15.5$ (for $V < 13.7$ km/sec) has been used.[3]

The total heating, per unit area, experienced by the body during the entry is

$$q = \int_0^t \frac{dq}{dt}\, dt \tag{5}$$

By combining Eqs. (1, 3, and 4) and substituting the result into Eq. (5)

$$q = \frac{CV_E^{M-1}}{\beta \sin\gamma_E} \int_0^1 (\bar{\rho})^{N-1} e^{-(B/2)(M-1)\bar{\rho}} d\bar{\rho} \tag{6}$$

Setting

$$k = (B/2)(M - 1)$$

and rewriting the integral

$$\int_0^1 (\bar{\rho})^{N-1} e^{-k\bar{\rho}}\, d\bar{\rho} = \int_0^\infty (\bar{\rho})^{N-1} e^{-k\bar{\rho}}\, d\bar{\rho} - \int_1^\infty (\bar{\rho})^{N-1} e^{-k\bar{\rho}}\, d\bar{\rho}$$

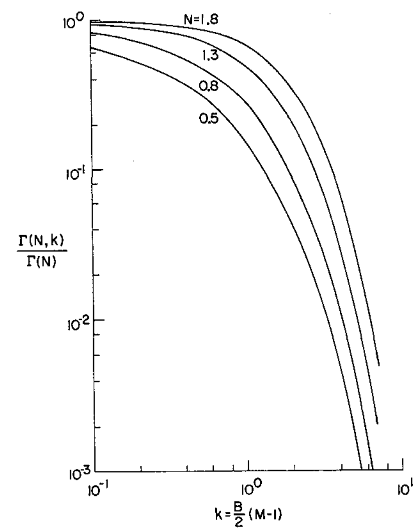

Fig. 1 Ratio of incomplete to complete gamma function.

Received January 26, 1970.

* Research Scientist. Associate Fellow AIAA.

where the first integral on the right-hand side can now be identified with the gamma function[4] and the second one with the incomplete gamma function, so that†

$$q = (CV_E^{M-1}k^{-N}/\beta \sin\gamma_E)[\Gamma(N) - \Gamma(N,k)] \quad (7)$$

Ratios of the incomplete to the complete gamma function are shown in Fig. 1 for several values of N; it can be seen that this ratio becomes small for k $\dot{>}$ 4. Physically, this corresponds to bodies which have decelerated to a small fraction of their entry velocity at impact. Thus, for $V_F \ll V_E$

$$q \doteq (CV_E^{M-1}k^{-N})/(\beta \sin\gamma_E)\Gamma(N) \quad (8a)$$

or, in terms of the parameters of primary interest

$$q \sim (m/C_DA)^N(\beta \sin\gamma_E)^{N-1} \quad (8b)$$

where only the exponent of the density, N, appears. When the body's final velocity is not small

$$q \sim (m/C_DA)^N(\beta \sin\gamma_E)^{N-1}[\Gamma(N) - \Gamma(N,k)] \quad (8c)$$

For the limiting case of a nearly constant velocity entry‡ ($V \doteq V_E$), it can easily be shown that for an exponential atmosphere

$$dt \doteq -dy/V_E \sin\gamma_E = d\bar{\rho}/\bar{\rho}\beta V_E \sin\gamma_E \quad (9)$$

Substituting Eqs. (4) and (9) into Eq. (5) and integrating yields

$$q \doteq CV_E^{M-1}/N\beta \sin\gamma_E \quad (10a)$$

or

$$q \dot{\sim} (\beta \sin\gamma_E)^{-1} \quad (10b)$$

The altitude and velocity of maximum heating can also readily be found by substituting Eq. (3) into (4), setting the derivative of Eq. (4), with respect to $\bar{\rho}$ equal to zero and solving, to yield

$$\bar{\rho} = 2N/MB \quad (11a)$$

and

$$V/V_E = e^{-N/M} \quad (11b)$$

The following conclusions are noted.

1) The total heat input is substantially reduced by decreasing m/C_DA, except in the limiting case of bodies which do not decelerate significantly during entry, when heating becomes independent of m/C_DA.

2) For a point on the body experiencing only laminar flow, such as the stagnation point, the total heating decreases significantly with increasing entry angle. (However, increasing the entry angle also increases the Re, since $Re \sim (m/C_DA)(\beta \sin\gamma_E)$ so that keeping γ_E and, therefore Re, small enough to preserve laminar flow, generally results in considerably less heat input, for bodies with $V_F \ll V_E$, than using steep entry angles if turbulent boundary-layer flow occurs at high speed.)[3] For laminar flow, the maximum heating rate occurs at $V/V_E \doteq 0.85$.

3) For areas on a body experiencing predominantly turbulent flow during entry, the total heat input still decreases with increasing entry angle, but much more slowly than for laminar flow. If transition to turbulence occurs early in the entry (roughly true if $B < 20$), then peak heating occurs at about $V/V_E \doteq 0.80$. However, for larger values of B, transition and, consequently, peak turbulent heating will occur nearer the speed for maximum Reynolds number which is $V/V_E \doteq 0.37$ or considerably later during the entry.

4) For bodies experiencing predominantly radiative heating, it appears that shallow entries always reduce the total heat input. Although the value of N can vary considerably with flight conditions, atmospheric composition, etc., it is probably always greater than one, for cases of practical interest. Peak radiative heating in air occurs roughly at $V/V_E \doteq 0.89$

† Because constant values of N and M were used during the integration leading to Eq. (7), only continuum flow heating processes have been considered. This is approximately valid for $B < 10^3/\sin\gamma_E$ and in general excludes only very small bodies, roughly of centimeter size or less.

‡ For instance, for a steep ($\sin\gamma_E \doteq 1$) Earth atmospheric entry such that $V_F = 0.9V_E$, a spherical iron body of 4m radius, or 5° half-angle cone 2.5m long, having a specific gravity of one, is required.

References

[1] Allen, H. J. and Eggers, A. J., "A Study of the Motion and Aerodynamic Heating of Missiles Entering the Earth's Atmosphere at Supersonic Speeds," TN 4047, 1957, Ames Lab., Moffett Field, Calif., NACA.

[2] Chapman, D. R., "An Approximate Analytical Method for Studying Entry into Planetary Atmospheres," TN 4276, 1958, Ames Lab., Moffett Field, Calif., NACA.

[3] Allen, H. J., Seiff, A., and Winovich, W., "Aerodynamic Heating of Conical Entry Vehicles at Speeds in Excess of Earth Parabolic Speed," TR R-185, 1963, NASA.

[4] Abramowitz, M. and Stegun, I., eds., *Handbook of Mathematical Functions*, National Bureau of Standards Applied Mathematics Series 55, Washington D.C., 1964.

Optimization of Search for an Object Drifting in Outer Space

Olavi Bertel Hellman*
University of California, Berkeley, Calif.

A SPACECRAFT that becomes unable to use its own power in outer space starts drifting and becomes a subject of search. We shall assume that the search for the drifting object is carried out by using equipment with limited range of detection. If the location and the velocity of the object at the moment when the drifting starts were known to the searchers, the future track of the object would be calculated accurately and the object would be located by using this information. In the present paper we shall consider the problem involved in optimizing the search for a drifting object whose location and speed at the moment when the drifting starts are known only approximately. Furthermore, we shall assume that the search may be expressed in terms of the so-called search density. An expression for the optimal search density will be derived.

Basic Equations

We shall assume that the location $\mathbf{x}(0)$ and the velocity $\mathbf{v}(0)$ of an object in outer space, at time 0, possess the probability densities $g[\mathbf{x}(0)]$ and $h[\mathbf{v}(0)]$, respectively, and that $\mathbf{x}(t)$, the location of the object at time t, satisfies a differential equation

$$\ddot{\mathbf{x}} = \mathbf{f}(\dot{\mathbf{x}},\mathbf{x}) \quad (1)$$

where the dot denotes the time derivative, which is solvable in a closed form or numerically. One now wishes to organize

Received Feburary 18, 1970.

* Professor of Applied Mathematics, Department of Industrial Engineering and Operations Research; on leave of absence from the University of Turku, Turku, Finland.